

Term Paper: The Hall and Taylor Model in Duali¹

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5/8/2012

1 Introduction

In macroeconomics and policy making arena, it is extremely important to have the ability to manipulate a set of control variables (like money stock and government expenditure) to influence the dynamics of an economic system, to finally achieve various policy targets (like the GDP growth rate, unemployment etc.). And Duali is a specialized computational tool to do such simulations. So in this term paper I will explore the Hall and Taylor model in Duali and learn how to do stochastic control for certain economic systems.

From Chapter 13 of the textbook, we know that the Hall and Taylor model is a “standard IS-LM-Open Economy submodel for the aggregate demand and of the economy together with the aggregate supply”², which has 12 equations. The Hall and Taylor model in Duali³ provides a clear and accessible specification in which we can easily study the effects of macroeconomic shocks and changes to exogenous policy variables through a small IS-LM-Open economy model. The model has a steady state and by Duali it always produces a resolution to macroeconomic shocks and returns to its initial steady state. This paper will discuss the model in various stochastic settings and analyze its policy implications.

The paper is organized as follows. Section 2 introduces the optimal policy analysis methods with Duali in a deterministic case. Section 3 discusses the model with parameter uncertainty, which is a form of stochastic term. Section 4 considers another type of stochastic sources, additive noise. Section 5 concludes.

¹ Most of the paper is from my weekly assignments. I would like to thank Professor David Kendrick for many helpful comments. And this paper is based heavily on Chapter 17 of Kendrick, Mercado and Amman (2006).

² Kendrick, Mercado and Amman (2006), P248.

³ This model was programmed for Duali by Dr. David Kendrick at the University of Texas and is based on the model from Hall and Taylor (1997).

2 Optimal Policy Analysis with Duali: a Deterministic Case

The Hall and Taylor (1997) model is a twelve-equation nonlinear dynamic model for open economy with flexible exchange rates, which can be seen from Chapter 13 of the textbook. We use Johansen's method to linearize the model to get a four-equation linear model⁴, which we solve and get the state-space form as in the textbook. With certain parameterization, we can solve the model in Duali and compare the results in different scenarios.

Without exogenous shocks in the deterministic case, the economy will stay in the steady state. All the variables in the model are the percent deviations from the steady state. So they will be zeroes if there is no shock.

The first experiment I want to do is change the initial value for the economy. Let $Y = -0.04$, $R = 0.02$, $plev = 0.05$ and $E = 0.03$. This state of economy is called stagflation if I recall correctly. People in this economy must be suffering. What I want to assess is how optimal paths for government expenditure (G) and the money supply differ from the autonomous paths. Firstly I need to solve the optimal paths in this initial value setup. Then I name this scenario and solve the autonomous paths by changing the weights in the criterion function⁵.

The results can be seen in Figure 1. Though many variables in the autonomous scenario change in a weird and hard-to-explain way. It is reassuring to see that the optimal paths for the states outperform the autonomous responses of the system for all four target variables.

Initially in stagflation, if no government policies to bring the economy to growth, it will display autonomous paths to go back to the steady-state. The real GDP is below the potential GDP, which will cause the price to fall. Though the price level was higher than the steady-state level in the initial period, it falls rapidly and even to be lower than the steady-state level. In this setup with interest rate and nominal exchange rate higher than the steady-state level initially, they go up in early periods. Gradually they go down as the price is falling. Then the investment and export will increase, which makes the real GDP gradually attain potential GDP.

⁴ The details can be found from the Appendix E of the textbook.

⁵ I do encounter a little problem here. Once we mistakenly name the scenarios we have no easy way to change it or delete it. My solution is to exit Duali and start the whole procedure again.

In the observed optimal path of GDP, we don't have the overshooting issue here as in the textbook (It does go to a little bit higher than the steady state but the magnitude is very tiny). The real GDP goes from 4 percent below steady state right back to its steady-state value, much faster than in the autonomous path.

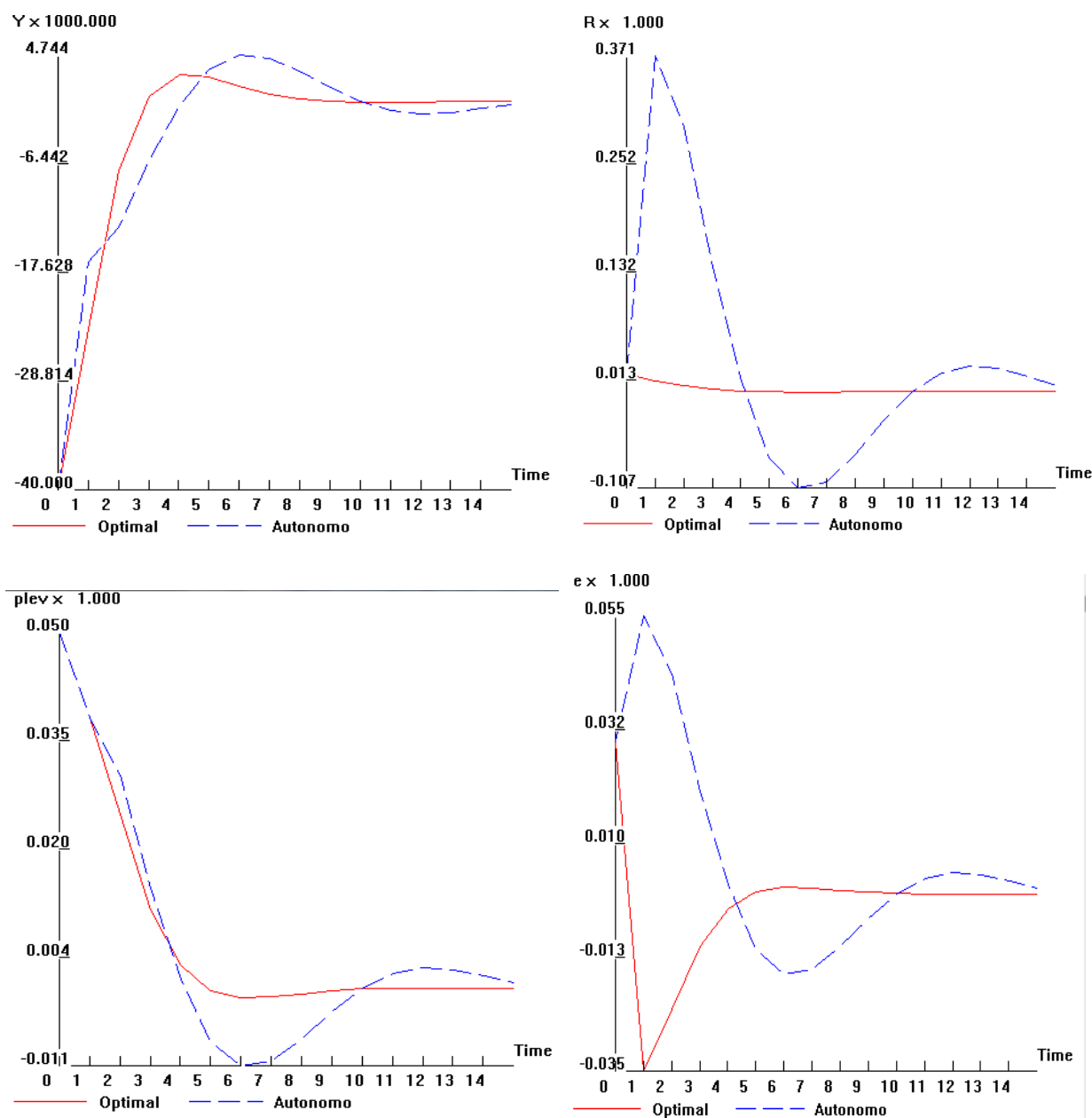


Figure 1 Autonomous response versus optimal control experiment

How is this good performance from? It can be attributed to the strange combination of monetary and fiscal policies, as can be seen in Figure 2. The government expenditure decreases 5.3 percent and the money supply keeps going up to 1.7 percent in the early

periods and then both policies go to the steady state level. Meanwhile as we can see from Figure 1, the interest rate does vary compared to a big increase of almost 37 percent implied by the autonomous paths of the system. The contracting fiscal policy exerts downward pressure on R and keeps it from going up. Finally it is back to its steady state level.

Again the monetary policy plays a minor role when compared to fiscal policy, as shown in Figure 2. We confirm that fiscal policy is much effective than monetary policy.

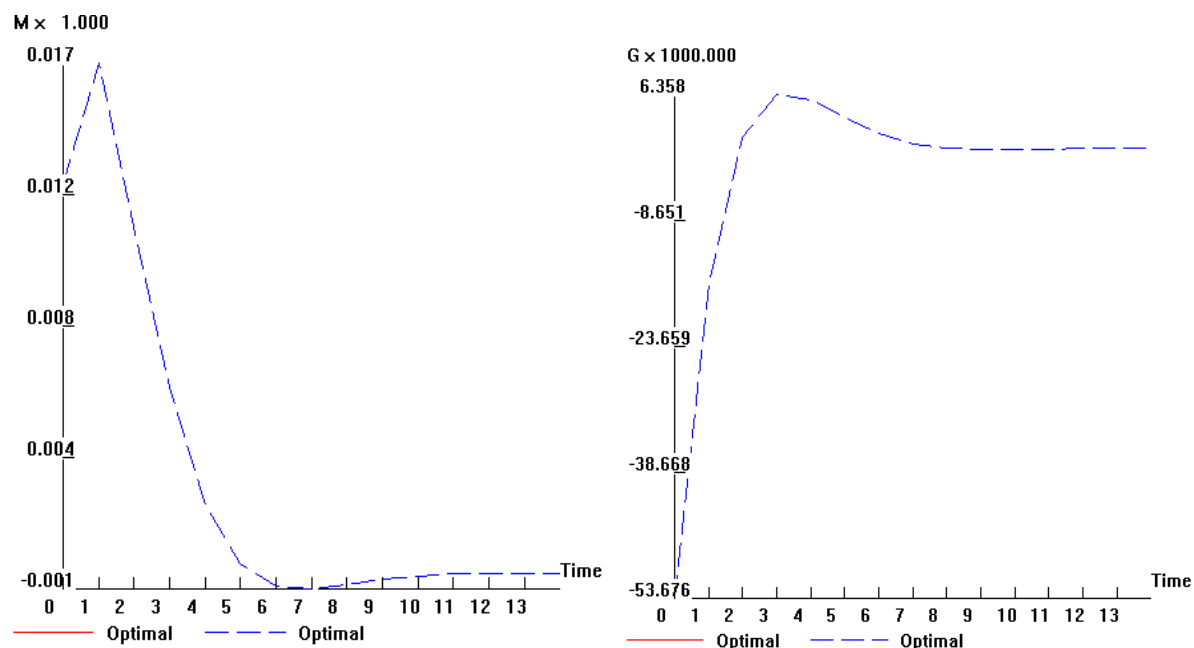


Figure 2: Optimal policy variables paths

Maybe it gives us some hints how to save our economy once we are in a stagflation. And with this model we can simulate lots of scenarios and find the optimal policy combinations of behavior of variables that policy maker can achieve given a calibration of the parameters. From this perspective, this model and the software Duali are very helpful for macroeconomic policy makers.

3 Stochastic Control: Parameter Uncertainty

In macroeconomics, taking uncertainty into account is interesting and extremely important for academic research and policy analysis. In stochastic setting, we typically consider three types of uncertainty: additive uncertainty, parameter uncertainty and

measurement error. It would be harder to explain and sell our results from models with these types of uncertainty, but still we can solve these models numerically and gain some insights for economic policy making.

As in the textbook, we want to compare different procedures for solving models with parameter uncertainty. In this experiment I will change the initial value for the economy and then compare the results from the standard QLP procedure and those from models with parameter uncertainty (OLF).

Consider an economy in stagflation, we change the initial values to be $Y = -0.04$, $R = 0.02$, $p_{lev} = 0.05$ and $E = 0.03$. What I want to assess first is how optimal paths for government expenditure (G) and the money supply using OLF differ with different initial values. To do this I need to solve the optimal paths using the stagflation setup first and then I ask scenario questions after solution and do this again with the original initial value, which is just $Y = -0.04$.

The results can be found in Figure 3. The insight from this figure is that policies should be very different, even opposite to address recession and stagflation. When there is just recession, we raise the government expenditure in the first period and then reduce it, reduce money supply in the first two periods and then increase money supply until the economy is back to the steady state. But for the stagflation, we lower the government expenditure until it is higher than the steady state and then reduce it to the steady state. For money supply we reduce it first and then reduce it further. From 3rd period we raise it until it is back to the steady state. It takes about the same time for optimal policy paths in these two scenarios.

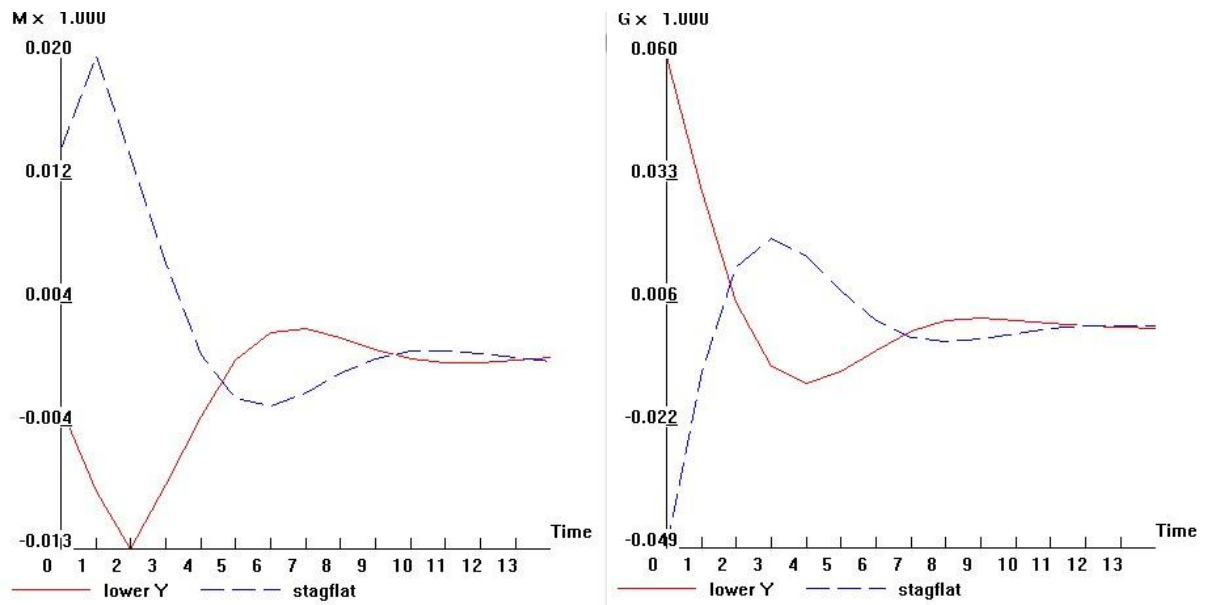


Figure 3: Optimal policy variables paths using OLF w/o update

Then we compare optimal policy variable paths in the stagflation setting, using deterministic QLP and stochastic OLF without update. In this comparison, the initial values are $Y = -0.04$, $R = 0.02$, $plev = 0.05$ and $E = 0.03$. As can be seen in the graphs in Figure 4, the dashed line represents results from QLP procedure and the solid line OLF w/o update.

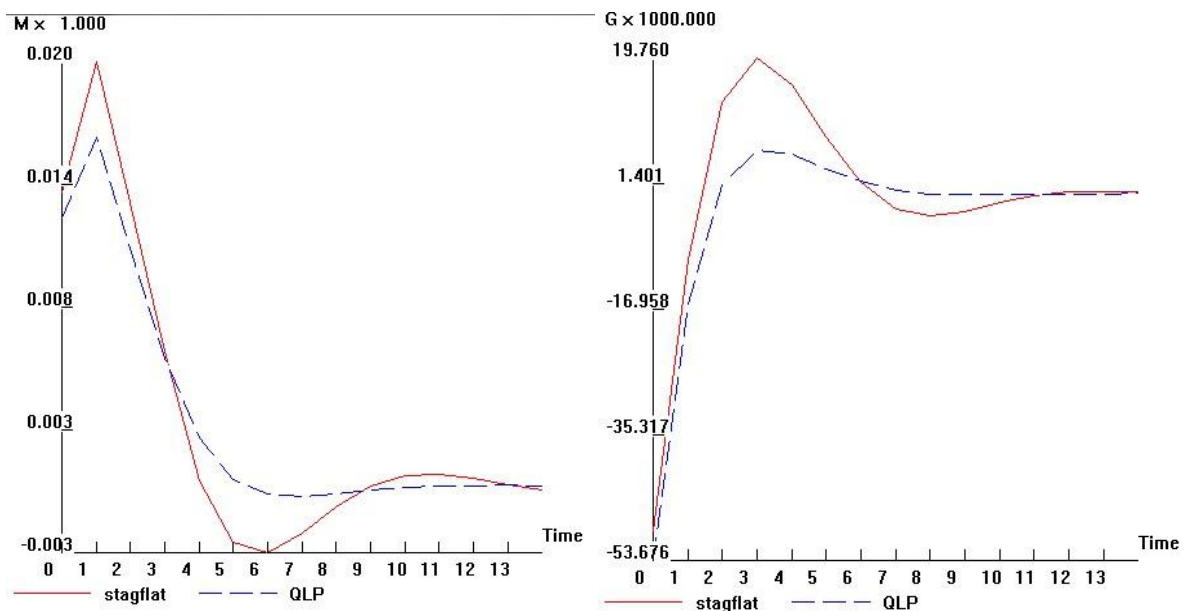


Figure 4: Optimal policy variables paths (QLP Vs OLF w/o update)

The use of both government expenditure and money supply are more aggressive with OLF w/o update, which contradicts the Brainard results mentioned in the textbook.

The next experiment I want to do is to increase the uncertainty for money supply and see whether it will be optimal to use monetary policy more cautiously. Basically assume we triple the standard deviation of parameters corresponding to money supply. So the SITTO matrix becomes

$$SITTO = \begin{pmatrix} 0.06741 & & & & & \\ & 0.00213 & & & & \\ & & 34.31376 & & & \\ & & & 0.76947 & & \\ & & & & 3.14677 & \\ & & & & & 0.04813 \end{pmatrix}$$

The graphs in Figure 5 contrast the behavior of policy variables for this experiment (OLF-B) against the behavior shown before in the OLF w/o update. As expected, the increase in the uncertainty of money supply parameters induces a more cautious use of monetary policy and a more aggressive fiscal policy during the first periods.

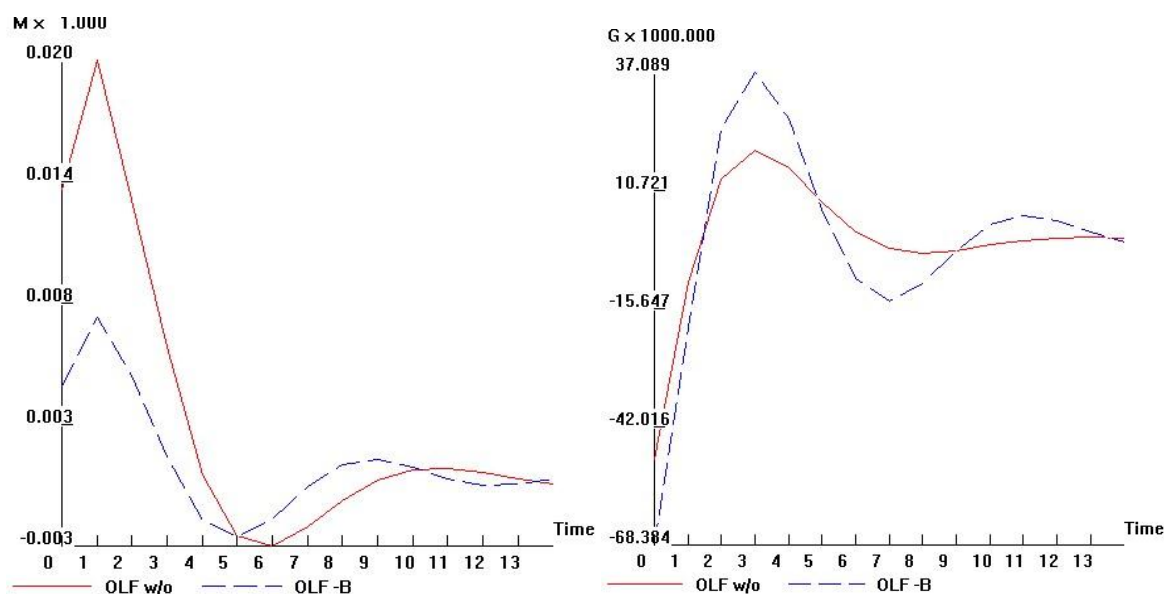


Figure 5: Optimal policy variables paths (increased uncertainty)

4 Stochastic Control: Additive Noise

In section 2 and section 3, I have done optimal policy analysis for the Hall and Taylor model using standard deterministic QLP procedure. In addition, I did some experiment of the model in the presence of parameter uncertainty and compared QLP and OLF without update.

Now we move to another type of stochastic sources. Consider there are random shocks hitting the economy every period, which can be modeled as additive noise in the Hall and Taylor model. As in the textbook, I will perform 100 Monte Carlo runs to contrast the results of QLP and OLF.

Let me sketch what is going on. We begin with the Hall and Taylor model. As discussed before, we have parameter uncertainty in the B matrix. Now we have some additive shock. In my experiment, GDP, price level, interest rate and nominal exchange rate have shocks with 10 percent standard deviation, which has more uncertainty than the example in the textbook. So we get the variance-covariance matrix of additive noises (Q) as follows:

$$Q = \begin{pmatrix} 0.01 & & & & & & & \\ & 0.01 & & & & & & \\ & & 0.01 & & & & & \\ & & & 0.01 & & & & \\ & & & & 0.1^{-9} & & & \\ & & & & & 0.1^{-9} & & \\ & & & & & & 0.1^{-9} & \\ & & & & & & & 0.1^{-9} & \\ & & & & & & & & 0.1^{-9} \end{pmatrix}$$

As discussed in the textbook, we get a realization of parameters at time zero and then get a vector of additive shocks very period. We compute optimal controls from period 1 to 14 by QLP or OLF and use the optimal control to compute the system forward. We will update the parameter estimates based on those realized ones and optimal controls by Kalman filter⁶.

⁶ The Kalman filter, as described in Wikipedia, also known as linear quadratic estimation (LQE), is an algorithm which uses a series of measurements observed over time, containing noise (random variations) and other inaccuracies, and produces estimates of unknown variables that tend to be more precise than those that would be

Then I did those Monte Carlo simulations in Duali. The results are saved in a debug file. Actually to open it we need to install Microsoft Visual Studio software. Anyway, I got the results:

Table 1 Monte Carlo Results

	CE	OLF
Average Criterion Value	97.6	96.62
Runs with Lowest Criterion	47	53

We can see from Table 1 the OLF procedure does slightly better than the CE, in term of average criterion value and number of runs with lowest criterion. Because no theory has been developed about the relative performance of QLP versus OLF, our computational Monte Carlo results here may give us some ideas about their performance.

By solving this model and doing the Monte Carlo runs, we could also compare the monetary policy and fiscal policy in this situation and those with only parameter uncertainty.

5 Conclusion and Further Expansions

In this term paper I did some experiments of the Hall and Taylor model in Duali. It gives us a good example how we can manipulate the state variables to get the desirable target variable results. However, in this model the economy always stays or goes back to its steady state. In reality, we may never be in the steady state and we never know what the equilibrium level of the economy would be. So it would be helpful if we can include more states of the economy in the model. For instance, we could add the long run growth to the model and measure the effects of exogenous forces in a growing economy. Also we could consider population growth. Wage can also be introduced as an endogenous variable in this system. We could also consider exchange rate as an important policy variable in this open economy model. These expansions can be done in

based on a single measurement alone. More formally, the Kalman filter operates recursively on streams of noisy input data to produce a statistically optimal estimate of the underlying system state. Details can be seen from http://en.wikipedia.org/wiki/Kalman_filter.

Duali, in a counterfactual framework or a Monte Carlo, as showed by Kendrick and Shoukry (2011, 2012).

Reference:

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